2.4 The Cosine Law

Learning Outcomes:

- Sketch a diagram and solve a problem using the cosine law
- Recognize when to use the cosine law to solve a given problem
- Explain the steps in the given proof of the cosine law

INVESTIGATE

One side of a right triangle is 8 cm. One angle is 50°. What could the other side lengths be?

KEY IDEAS!

- The cosine law can be used to determine an unknown _____ length or ______ measure in an acute triangle

\[
\begin{align*}
a^2 &= b^2 + c^2 - 2bc \cos A \\
b^2 &= a^2 + c^2 - 2ac \cos B \\
c^2 &= a^2 + b^2 - 2ab \cos C
\end{align*}
\]
Example 1: Determine the unknown side length to the nearest tenth of a cm.

\[ a^2 = b^2 + c^2 - 2 \cdot b \cdot c \cdot \cos A \]

\[ a^2 = (15^2) + (18)^2 - 2(15)(18) \cos 46 \]

\[ \sqrt{a^2} = \sqrt{173.88} \]

\[ a = 13.2 \text{ cm} \]
TRY ME!!! Determine the unknown side length to the nearest tenth of a cm.

\[ a^2 = b^2 + c^2 - 2bc \cos A \]
\[ a^2 = 9.5^2 + 10.5^2 - 2(9.5)(10.5)\cos 40 \]
\[ \sqrt{a^2} = \sqrt{47.67} \]
\[ a = 6.9 \text{ cm} \]
Example 2: Determine the angle $R$ to the nearest whole number

\[
\cos R = \frac{b^2 + c^2 - a^2}{2bc} \quad \cos A = \frac{b^2 + c^2 - a^2}{2bc} \\
\cos R = \frac{p^2 + q^2 - r^2}{2pq} \\
\cos R = 0.9289\ldots \\
\cos^{-1}(0.9289\ldots) = 21.7^\circ \Rightarrow 22^\circ
\]
TRY ME!!! Determine the angle $\angle Z$ to the nearest whole number

\[
\cos A = \frac{b^2 + c^2 - a^2}{2bc}
\]

\[
\cos A = \frac{2.6^2 + 2.2^2 - 2.9^2}{2(2.6)(2.2)}
\]

\[
\cos A = \frac{3.19}{11.44}
\]

\[
\cos^{-1}\left(\frac{3.19}{11.44}\right) = 73.8^{\circ}
\]

\[
A = 74^{\circ}
\]
Example 3: A three-pointed star is made up of an equilateral triangle and three congruent isosceles triangles. Determine the length of each side of the equilateral triangle in this three-pointed star. Round the length to the nearest centimeter.

\[ a^2 = b^2 + c^2 - 2bc \cos A \]

\[ a^2 = 60^2 + 60^2 - 2(60)(60)(\cos 20) \]

\[ a^2 = 434.213... \]

\[ a = 20.8 \text{ cm} \]

Assignment: pg. 119-125 #1-5, 7, 9, 10, 12, 15, 18, 21, 23, 26, 30